An Efficient Dynamic Analysis Using Hertz Model for Vehicle-Track Interaction on Top of Unsupported Multi-layered Ballasted Track

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The ballasted track is a potential type of track in the railway engineering. Basically, many numerical simulation methods have modeled the vehicle-track interaction. The ballasted track is a complex structure composed of rail, rail pad, sleeper and ballast placed on subgrade. Several models have been proposed for modeling the vehicle-track interaction analysis. In this paper, the vehicle is modeled as quarter-bogie moving on top of rail that is simulated as Timoshenko beam. Rail is supported by sleeper and rail pad with vertical stiffness and damping. Especially, rail has an unsupported sleeper at the middle of the track. A new improved ballast model is adapted in this study to consider the longitudinal continuity of the granular layers on ballasted track. The interaction between wheel and rail defined by a special vector is considered as a contact point vector. It allows representing the time-variant stiffness matrix over time. As a result, as well as the contact force could be obtained, change of contact force when a vehicle passing unsupported sleeper was obtained. It could be also clarified what efficient ballast continuous model be adapted for case of unsupported sleeper.

**Keyword:** Vehicle-track dynamic interaction, ballast model, quarter-bogie model, unsupported sleeper.

1. Introduction

For vehicle-track interaction analysis many physical and structural parameters must be considered. In this study, rail is modeled as Timoshenko beam and is incorporated to FEM method. A new improved ballast model is adapted in this study to consider the longitudinal continuity of the granular layers on ballasted track. The vehicle is modeled as quarter-bogie moving with constant speed on top of rail that is simulated as Timoshenko beam. Especially, rail has an unsupported sleeper at the middle of the track.

2. Dynamic Model

2.1 Track model

There is a large volume of published studies for rail models such as Sun et al. (2002), Abe et. al. (1998) and Yang (2009).

2.2 Vehicle model

The quarter – bogie model consists of a wheel set with bogies masses that are connected to primary suspensions represented by linear springs and dampers in parallel. The model has two degrees of freedom (DOF) that consider vertical displacement of the wheelset and vertical displacement of the bogie. This means that rotations of the vehicle about longitudinal and transverse axis are ignored.

2.3 Ballast model

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Many ballast models have been developed such as Lei and Noda (2002). An improved model was suggested by Sun and Dhanasekar (2002) (Fig. 2) and was adapted for this study. A special pyramid model was adapted to consider longitudinal continuity of the granular layers.

**Fig. 2** Vehicle-track interaction model: a. Quarter-bogie model, b. Track foundation model Sun and Dhanasekar (2002)

### 2.4 Dynamic equation of the vehicle-track model

The governing equation is simply represented by using the well-known matrix form as

\[
[M]\ddot{u} + [C] \dot{u} + [K]u = f(t)
\]

where \(u\) is the displacement vector of all unknown variable in the model, \([M]\), \([C]\) and \([K]\) are the mass, damping and stiffness matrices respectively. The vector \(f(u,t)\) represents the non-linear dynamic contact force that is expressed by the following special defined vector, \(a(x_w,t)\). Assuming the wheel is traveling on the \(i^{th}\) element of the rail, the moving vector \(a(x_w,t)\) is obtained by

\[
a(x_w,t) = [0, K_1(x_1), N_2(x_1), N_3(x_1), N_4(x_1), K_2]^{\top}
\]

\[
f(t) = B_0(0 \ 1)^{\top} + F(t)(-1 \ a)^{\top}
\]

**3. Example**

Simulation of vehicle-track interaction modeled as quarter-bogie model moving on top of rail was performed by assuming the rail as Timoshenko beam that was supported by track foundation with sleeper, ballast, and subgrade. The mass of bogie is 3200 kg, the stiffness and damping of suspension are 2.08E+6 N/m and 1.00E+5 Ns/m respectively. The mass of wheel is 2400 kg and the wheel-rail contact stiffness is 1.33E+9 MN/m. The rail UIC60 rail is modeled as Timoshenko beam supported by sleeper with ballast at sleeper intervals of 0.6 m. The mass of sleeper is 340 kg.

The stiffness and damping of railpad are 8.00E+7 N/m and 5.00E+4 Ns/m, respectively. Vertical stiffness and damping of the ballast modeled as longitudinal continuity of the granular layers are 1.20E+8 N/m and 6.00E+4 Ns/m, respectively. The vertical stiffness and damping of subgrade are 6.00E+7 N/m and 9.00E+4 Ns/m, respectively. The unsupported position of the track is located at the middle of the length of rail with a gap sleeper. Some of continuity model of the ballast according to Sun and Dhanasekar (2002) was modified for the unsupported sleeper.

Fig. 3 illustrates the contact force calculated by the Newmark direct integration method. The contact force was fluctuated when a vehicle passing on the point of unsupported location. Contact force has a sine function simply because rail profile was without irregularities. The contact force decline from the adjacent before the unsupported position and remained minimum. After passing the position, contact force increase drastically and was fluctuated through some adjacent sleeper before becoming steady state.

**Fig. 3** The contact force when a vehicle passing unsupported position

### 4. Conclusion

This paper presents a new approach for calculation of contact force in case of unsupported position of the track and would be improved with irregularities in the future.

**Reference**


